27. Fourier Transform Properties

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$$x\left(\frac{t}{\tau}\right) \Leftrightarrow |\tau| X(\tau\omega)$$

$$x(t \pm t_0) \Leftrightarrow X(\omega) e^{\pm j\omega t_0}$$

$$X(t) \Leftrightarrow 2\pi\, x(-\omega)$$

If you know (via Fourier transform integral *or* you memorized) that

$$\mathcal{F}\{\text{rect}(t)\} = \text{sinc}\left(\frac{\omega}{2\pi}\right)$$

then there is no need to integrate again to find the Fourier transform of variants of this signal, such as

$$\mathcal{F}\{A\ \text{rect}(t)\}$$

$$\mathcal{F}\{A\ \text{rect}(t - t_0)\}$$

$$\mathcal{F}\left\{A\ \text{rect}\left(\frac{t - t_0}{\tau}\right)\right\}$$

Just apply Fourier transform **properties** on the basic solution to obtain the new answers.

Property	$\boldsymbol{x(t)}$	$\boldsymbol{X(\omega) = \mathcal{F}\{x(t)\}}$
Linearity (superposition)	$a\,x(t) + b\,y(t)$	$a\,X(\omega) + b\,Y(\omega)$
Complex conjugate	$x^*(t)$	$X^*(-\omega)$
Time scaling (reciprocal spreading)	$x\left(\frac{t}{\tau}\right)$	$\lvert\tau\rvert\,X(\tau\omega)$
Time inversion (time reversal)	$x(-t)$	$X(-\omega)$
Time shift (time delay/advance)	$x(t \pm t_0)$	$X(\omega)e^{\pm j\omega t_0}$
Frequency shift	$x(t)e^{\pm j\omega_0 t}$	$X(\omega \mp \omega_0)$
Duality	$X(t)$	$2\pi\,x(-\omega)$
Time convolution	$x(t) * y(t)$	$X(\omega)Y(\omega)$
Frequency convolution	$x(t)\,y(t)$	$\frac{1}{2\pi}X(\omega) * Y(\omega)$
Time differentiation	$\frac{d^n}{dt^n}x(t)$	$(j\omega)^n\,X(\omega)$
Time integration	$\int_{-\infty}^{t} x(\tau)d\tau$	$X(\omega)/(j\omega) + \pi X(0)\,\delta(\omega)$

Linearity (superposition) property

If we have for two signals $x(t) \Leftrightarrow X(\omega)$ and $y(t) \Leftrightarrow Y(\omega)$, then

$$a\,x(t) + b\,y(t) \Leftrightarrow a\,X(\omega) + b\,Y(\omega)$$

where a and b are constants.

Proof.

$$\mathcal{F}\{a\,x(t) + b\,y(t)\} = \int_{-\infty}^{\infty} [a\,x(t) + b\,y(t)]e^{-j\omega t}dt$$

$$= a\int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt + b\int_{-\infty}^{\infty} y(t)e^{-j\omega t}dt = a\,X(\omega) + b\,Y(\omega)$$

Superposition applies to any finite number of added signals. It also applies to an infinite number of added signals if we have enough conditions to interchange the summation and integration.

Q1. If you calculated that $\mathcal{F}\{u(t)\} = \pi\,\delta(\omega) + \frac{1}{j\omega}$, determine $\mathcal{F}\{\text{sgn}(t)\}$.

Q1. Solution. The definition of $\text{sgn}(t)$ is

$$\text{sgn}(t) = 2u(t) - 1$$

Hence,

$$\mathcal{F}\{\text{sgn}(t)\} = \mathcal{F}\{2u(t) - 1\} = 2\mathcal{F}\{u(t)\} - \mathcal{F}\{1\}$$
$$= 2 \times \left[\pi\,\delta(\omega) + \frac{1}{j\omega}\right] - 2\pi\,\delta(\omega) = \frac{2}{j\omega}$$

Complex conjugate property

If we have $x(t) \Leftrightarrow X(\omega)$, then

$$x^*(t) \Leftrightarrow X^*(-\omega)$$

where $x(t)$ is any general complex-valued signal. If $x(t)$ is the special case of real-valued signal, then

$$X(\omega) = \mathcal{F}\{x(t)\} = \mathcal{F}\{x^*(t)\} = X^*(-\omega)$$

Hence, for real-valued signal $x(t)$

$$X(\omega) = X^*(-\omega)$$

$$|X(\omega)| = |X(-\omega)| \quad [\textit{even sym.}] \quad \& \quad \angle X(\omega) = -\angle X(-\omega) \quad [\textit{odd sym.}]$$

Time scaling (reciprocal spreading) property

If we have $x(t) \Leftrightarrow X(\omega)$, then

$$x\left(\frac{t}{\tau}\right) \Leftrightarrow |\tau|\, X(\tau\omega)$$

If τ is a positive real number (say $\tau = 5$ seconds), we have

$$x\left(\frac{t}{\tau}\right) \Leftrightarrow \tau\, X(\tau\omega)$$

Similarly,

$$x(\lambda\, t) \Leftrightarrow \frac{1}{|\lambda|}\, X\left(\frac{\omega}{\lambda}\right)$$

Proof. For a real positive constant $\tau > 0$, we can perform a change of variables ($\xi = t/\tau$ and $d\xi = dt/\tau$) in the following integral

$$\mathcal{F}\{x(t/\tau)\} = \int_{t=-\infty}^{t=\infty} x(t/\tau)e^{-j\omega t}dt = \tau \int_{\xi=-\infty}^{\xi=\infty} x(\xi)e^{-j(\tau\omega)\xi}d\xi = \tau\, X(\tau\omega)$$

For a real negative constant $\tau < 0$, use ($\xi = t/\tau$ and $d\xi = dt/\tau$), and notice that the limits of integration reverse because of the substitution

$$\mathcal{F}\{x(t/\tau)\} = \int_{t=-\infty}^{t=\infty} x(t/\tau)e^{-j\omega t}dt = \tau \int_{\xi=\infty}^{\xi=-\infty} x(\xi)e^{-j(\tau\omega)\xi}d\xi = -\tau\, X(\tau\omega)$$

Notice that reversing the limits added the negative sign.

Q2. For $x(t) = 5\ \mathrm{rect}\left(\frac{t}{30}\right)$, determine $X(\omega) = \mathcal{F}\{x(t)\}$.

Q2. Solution. We know from the Fourier transform integral (see earlier examples) that (*memorize*)

$$\mathcal{F}\{\mathrm{rect}(t)\} = \mathrm{sinc}\left(\frac{\omega}{2\pi}\right)$$

Hence,

$$\mathcal{F}\left\{A\ \mathrm{rect}\left(\frac{t}{\tau}\right)\right\} = A\,\mathcal{F}\left\{\mathrm{rect}\left(\frac{t}{\tau}\right)\right\} = A\ |\tau|\ \mathrm{sinc}\left(\frac{\omega\tau}{2\pi}\right)$$

Then,

$$\mathcal{F}\left\{5\ \mathrm{rect}\left(\frac{t}{30}\right)\right\} = 5 \times 30 \times \mathrm{sinc}\left(\frac{30\omega}{2\pi}\right) = 150\ \mathrm{sinc}\left(\frac{15\omega}{\pi}\right)$$

The property of reciprocal spreading means that:

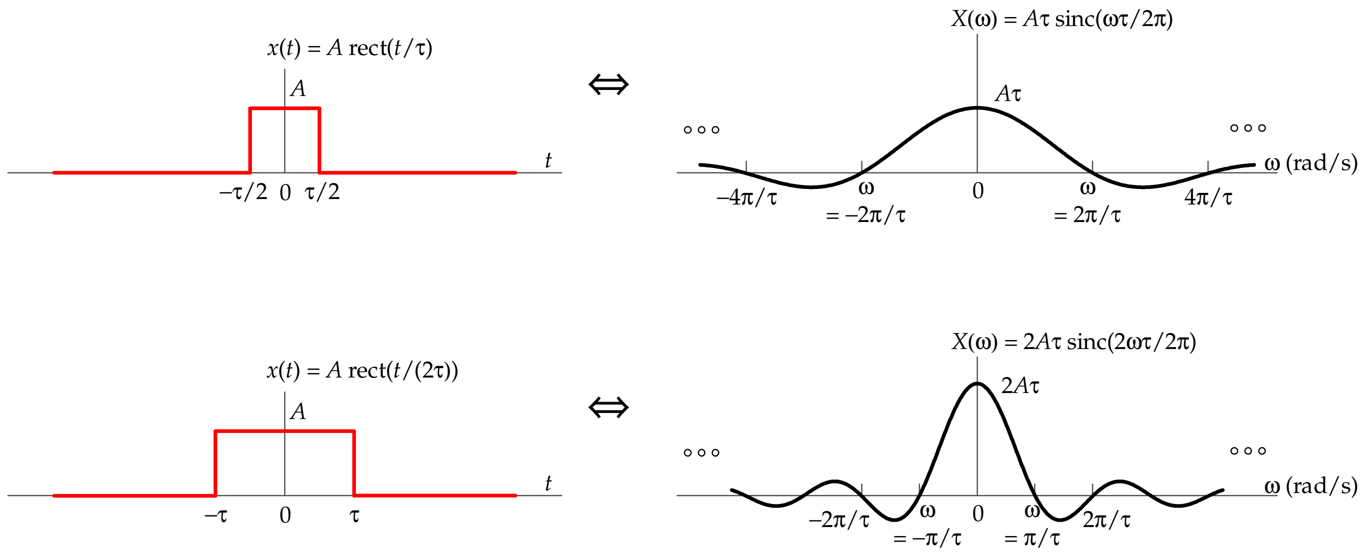
Compression of the signal in time-domain results in **expansion** of its Fourier transform (in frequency-domain).

Also,

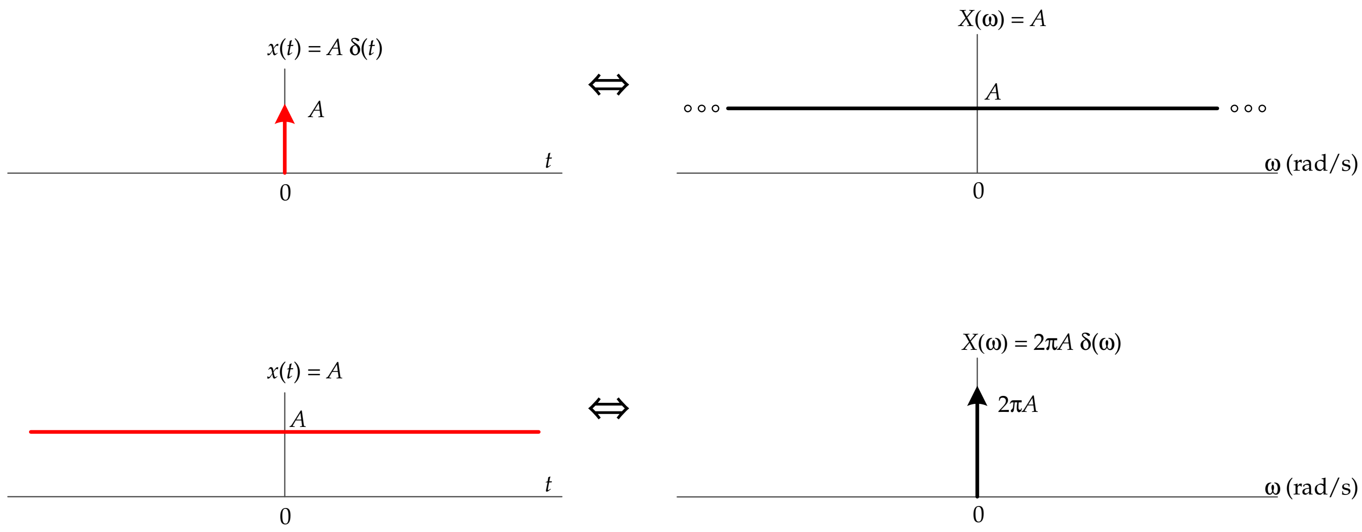
Expansion of the signal in time-domain results in **compression** of its Fourier transform (in frequency-domain).

Another way to look at it:

The wider the signal pulse in time-domain, the smaller the bandwidth of the signal (calculated from frequency-domain).



x(t) = A rect(t/τ)
A
t
−τ/2 0 τ/2
⇔
X(ω) = Aτ sinc(ωτ/2π)
Aτ
ω (rad/s)
−4π/τ
ω = −2π/τ
0
ω = 2π/τ
4π/τ
x(t) = A rect(t/(2τ))
A
t
−τ 0 τ
⇔
X(ω) = 2Aτ sinc(2ωτ/2π)
2Aτ
ω (rad/s)
−2π/τ
ω = −π/τ
0
ω = π/τ
2π/τ



x(t) = A δ(t)
A
t
0
⇔
X(ω) = A
A
ω (rad/s)
0
x(t) = A
A
t
0
⇔
X(ω) = 2πA δ(ω)
2πA
ω (rad/s)
0

Time inversion (or time reversal or time reflection) **property** can be obtained from the time scaling property. Time inversion causes frequency inversion. Remember that,

$$x\left(\frac{t}{\tau}\right) \Leftrightarrow |\tau|\, X(\tau\omega)$$

By letting $\tau = -1$

$$x\left(\frac{t}{-1}\right) \Leftrightarrow |-1|\, X(-1 \times \omega)$$

Hence,

$$x(-t) \Leftrightarrow X(-\omega)$$

Q3. You already calculated that

$$\mathcal{F}\{e^{-at}u(t)\} = \frac{1}{a + j\omega}$$

for $a > 0$. Determine $\mathcal{F}\{e^{at}u(-t)\}$.

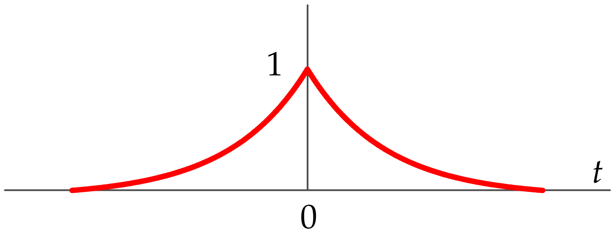
Q3. Solution. Using

$$x(-t) \Leftrightarrow X(-\omega)$$

we get,

$$\mathcal{F}\{e^{at}u(-t)\} = \frac{1}{a + j(-\omega)} = \frac{1}{a - j\omega}$$

Q4. Determine $\mathcal{F}\{e^{-a|t|}\}$ for $a > 0$.



Q4. Solution.

$$e^{-a|t|} = e^{-at}u(t) + e^{at}u(-t)$$

We know that

$$\mathcal{F}\{e^{-at}u(t)\} = \frac{1}{a+j\omega} \quad \text{and} \quad \mathcal{F}\{e^{at}u(-t)\} = \frac{1}{a-j\omega}$$

Hence,

$$\mathcal{F}\{e^{-a|t|}\} = \mathcal{F}\{e^{-at}u(t)\} + \mathcal{F}\{e^{at}u(-t)\} = \frac{1}{a+j\omega} + \frac{1}{a-j\omega} = \frac{2a}{a^2+\omega^2}$$

Time shift (time delay or time advance) property

If we have $x(t) \Leftrightarrow X(\omega)$, then

$$x(t - t_0) \Leftrightarrow X(\omega)e^{-j\omega t_0}$$

Also,

$$x(t + t_0) \Leftrightarrow X(\omega)e^{+j\omega t_0}$$

Usually written as

$$x(t \pm t_0) \Leftrightarrow X(\omega)e^{\pm j\omega t_0}$$

$$x(t \pm t_0) \Leftrightarrow |X(\omega)|e^{j\angle X(\omega)}e^{\pm j\omega t_0} = |X(\omega)|e^{j(\angle X(\omega) \pm \omega t_0)}$$

Q5. Compare the Fourier transform for $A\ \text{rect}\left(\frac{t}{\tau}\right)$ versus the Fourier transform for $x(t) = A\ \text{rect}\left(\frac{t-(\tau/2)}{\tau}\right)$.

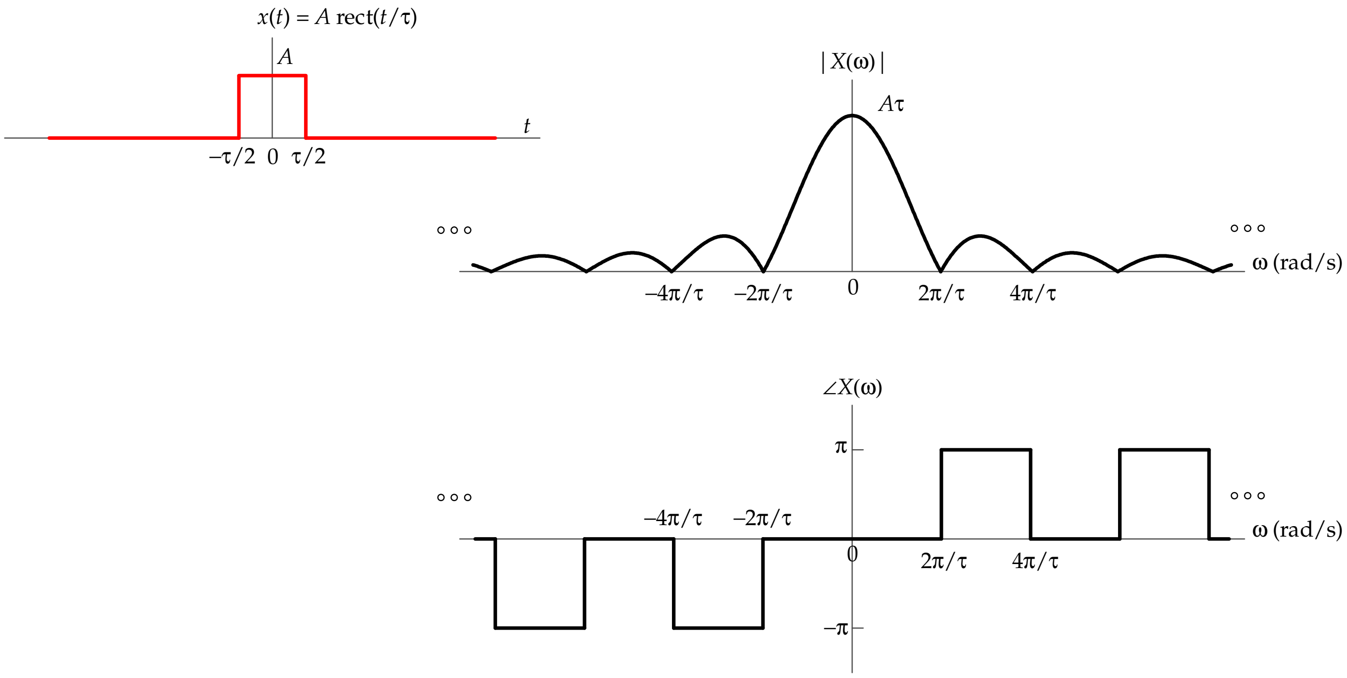
Q5. Solution. From an earlier example

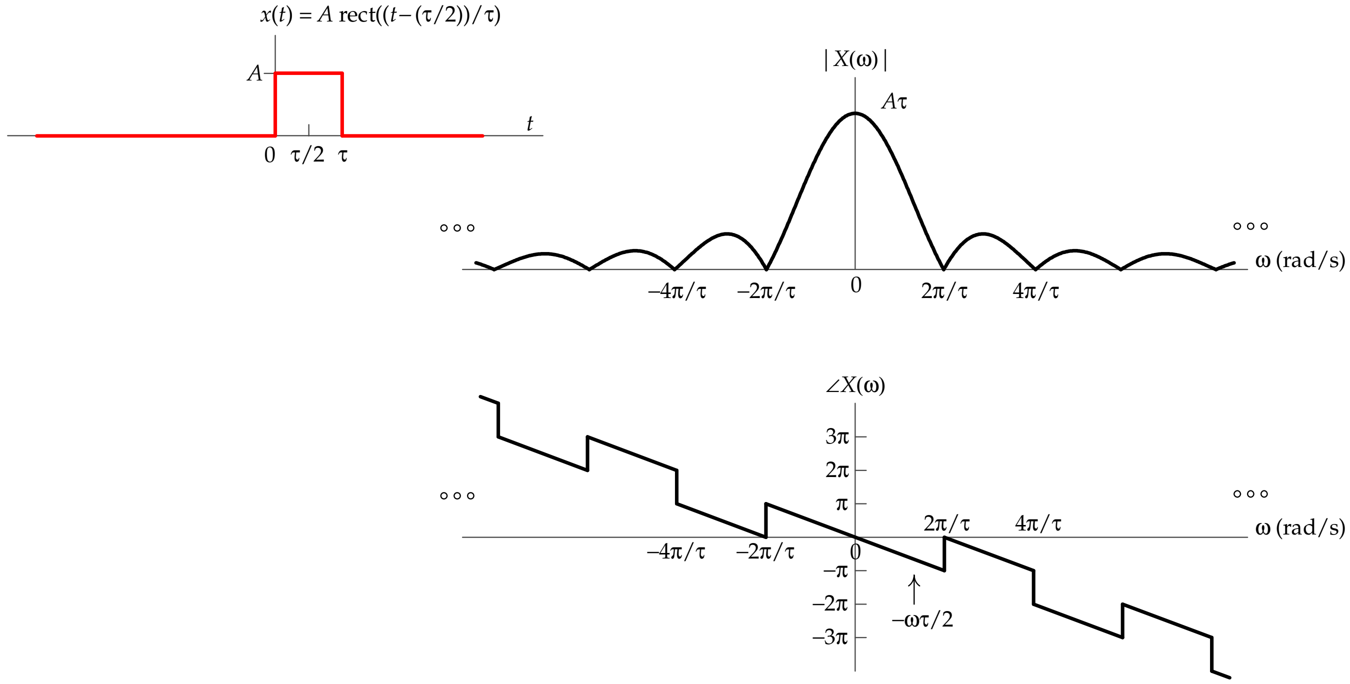
$$\mathcal{F}\left\{A\ \text{rect}\left(\frac{t}{\tau}\right)\right\} = A\,|\tau|\ \text{sinc}\left(\frac{\omega\tau}{2\pi}\right)$$

Hence,

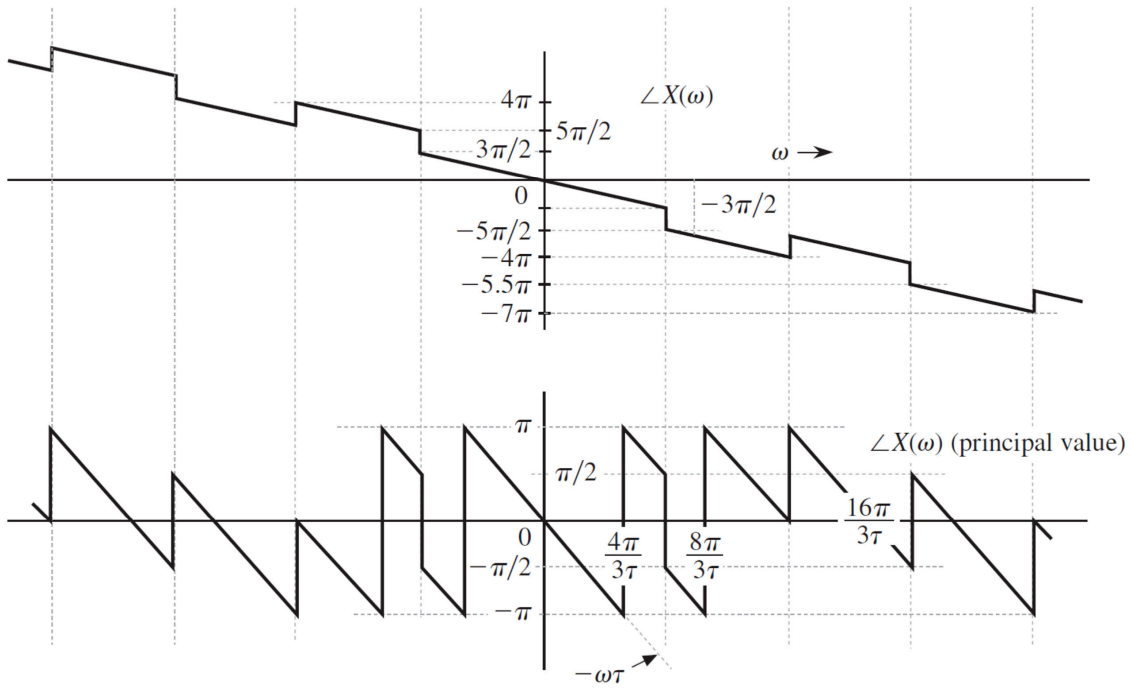
$$\mathcal{F}\left\{A\ \text{rect}\left(\frac{t-(\tau/2)}{\tau}\right)\right\} = A\,|\tau|\ \text{sinc}\left(\frac{\omega\tau}{2\pi}\right)e^{-j\omega(\tau/2)}$$

Hence, delaying a signal by t_0 seconds **does not change** its amplitude spectrum density, just its phase spectrum density (add linear line $-\omega t_0$)





x(t) = A rect((t−(τ/2))/τ)
A
t
0 τ/2 τ
| X(ω) |
Aτ
ω (rad/s)
−4π/τ −2π/τ 0 2π/τ 4π/τ
∠X(ω)
3π
2π
π
−π
−2π
−3π
−ωτ/2



∠X(ω)
4π
5π/2
3π/2
ω →
0
−3π/2
−5π/2
−4π
−5.5π
−7π
∠X(ω) (principal value)
π
π/2
−π/2
−π
4π/3τ
8π/3τ
16π/3τ
−ωτ

Frequency shift property

If we have $x(t) \Leftrightarrow X(\omega)$, then

$$x(t)e^{j\omega_0 t} \Leftrightarrow X(\omega - \omega_0)$$

Also,

$$x(t)e^{-j\omega_0 t} \Leftrightarrow X(\omega + \omega_0)$$

Typically written as

$$x(t)e^{\pm j\omega_0 t} \Leftrightarrow X(\omega \mp \omega_0)$$

This is a shift in the frequency-domain.

Q6. Determine $\mathcal{F}\{x(t)\cos(\omega_0 t)\}$, which is the well-known amplitude modulated signal.

Q6. Solution. Expanding $\cos(\omega_0 t)$ using Euler's identity, we get,

$$\begin{aligned}
\mathcal{F}\{x(t)\cos(\omega_0 t)\} &= \mathcal{F}\left\{x(t)\left[\frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2}\right]\right\} \\
&= \mathcal{F}\left\{\frac{1}{2}x(t)e^{j\omega_0 t} + \frac{1}{2}x(t)e^{-j\omega_0 t}\right\} \\
&= \frac{1}{2}\mathcal{F}\left\{x(t)e^{j\omega_0 t}\right\} + \frac{1}{2}\mathcal{F}\left\{x(t)e^{-j\omega_0 t}\right\} \\
&= \frac{1}{2}X(\omega - \omega_0) + \frac{1}{2}X(\omega + \omega_0)
\end{aligned}$$

Q7. If $x(t) = \text{rep}_{T_0}\{p(t)\}$ is a periodic signal with period T_0, determine its Fourier transform.

Q7. Solution. Since $x(t) = \text{rep}_{T_0}\{p(t)\}$ is periodic, it can be written using the complex exponential Fourier series form as:

$$x(t) = \text{rep}_{T_0}\{p(t)\} = \sum_{n=-\infty}^{\infty} \alpha_n\, e^{jn\omega_0 t}\,, \qquad \omega_0 = \frac{2\pi}{T_0}$$

Hence,

$$X(\omega) = \mathcal{F}\{x(t)\} = \mathcal{F}\left\{\sum_{n=-\infty}^{\infty} \alpha_n\, e^{jn\omega_0 t}\right\} = \sum_{n=-\infty}^{\infty} \mathcal{F}\{\alpha_n\, e^{jn\omega_0 t}\}$$

$$X(\omega) = \sum_{n=-\infty}^{\infty} \mathcal{F}\{\alpha_n\, e^{jn\omega_0 t}\} = \sum_{n=-\infty}^{\infty} 2\pi\alpha_n \delta(\omega - n\omega_0)$$

Hence, for **periodic** signals the Fourier series complex exponential form α_n can be converted into Fourier transform $X(\omega)$ by multiplying each α_n value by a factor of 2π, and converting each discrete value α_n into an impulse.

Q8. Determine the Fourier transform of an impulse train $x(t) = \text{rep}_{T_s}\{\delta(t)\} = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$.

Q8. Solution. $\text{rep}_{T_s}\{\delta(t)\} \Leftrightarrow \omega_s \text{rep}_{\omega_s}\{\delta(\omega)\}$ where $\omega_s = \frac{2\pi}{T_s}$.

Duality property

If we have $x(t) \Leftrightarrow X(\omega)$, then

$$X(t) \Leftrightarrow 2\pi\, x(-\omega)$$

Hence, if the Fourier transform of rect( ) is sinc( ), then the Fourier transform of sinc( ) is rect( ).

If the Fourier transform of an impulse is constant, then the Fourier transform of constant is an impulse, etc.

Q9. Determine $\mathcal{F}\{\text{sinc}(t)\}$ and $\mathcal{F}\{A\tau\,\text{sinc}(\tau\, t/2\pi)\}$.

Q9. Solution. We know from the Fourier transform integral (see earlier examples) that (*memorize*)

$$x(t) = \text{rect}(t) \Leftrightarrow X(\omega) = \text{sinc}\left(\frac{\omega}{2\pi}\right)$$

Hence,

$$X(t) = \text{sinc}\left(\frac{t}{2\pi}\right) \Leftrightarrow 2\pi\, x(-\omega) = 2\pi\, \text{rect}(-\omega)$$

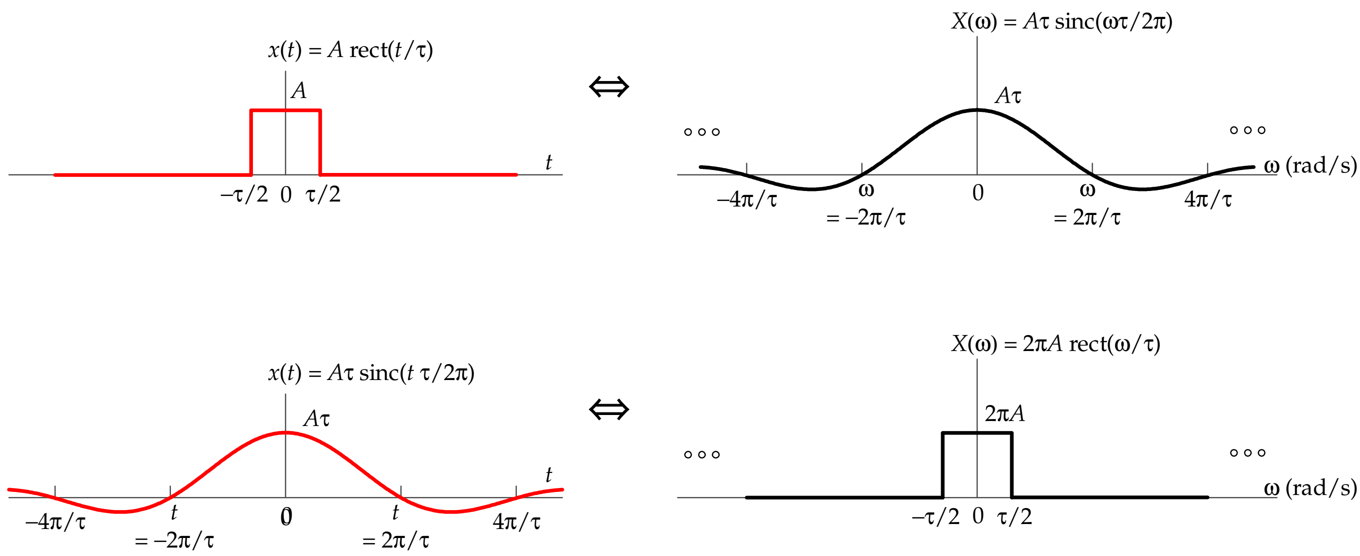
$$\text{sinc}(t) = \text{sinc}\left(2\pi \times \frac{t}{2\pi}\right) \Leftrightarrow \frac{2\pi}{2\pi}\, \text{rect}\left(\frac{-\omega}{2\pi}\right) = \text{rect}\left(\frac{\omega}{2\pi}\right)$$

$$\text{sinc}(t) \Leftrightarrow \text{rect}\left(\frac{\omega}{2\pi}\right)$$

$$\text{sinc}(\tau\, t/2\pi) \Leftrightarrow \left|\frac{2\pi}{\tau}\right| \text{rect}\left(\frac{2\pi}{\tau} \times \frac{\omega}{2\pi}\right)$$

$$A\tau\, \text{sinc}(\tau\, t/2\pi) \Leftrightarrow A\tau \times \frac{2\pi}{\tau} \times \text{rect}\left(\frac{2\pi}{\tau} \times \frac{\omega}{2\pi}\right)$$

$$A\tau\, \text{sinc}(\tau\, t/2\pi) \Leftrightarrow 2\pi A\, \text{rect}\left(\frac{\omega}{\tau}\right)$$



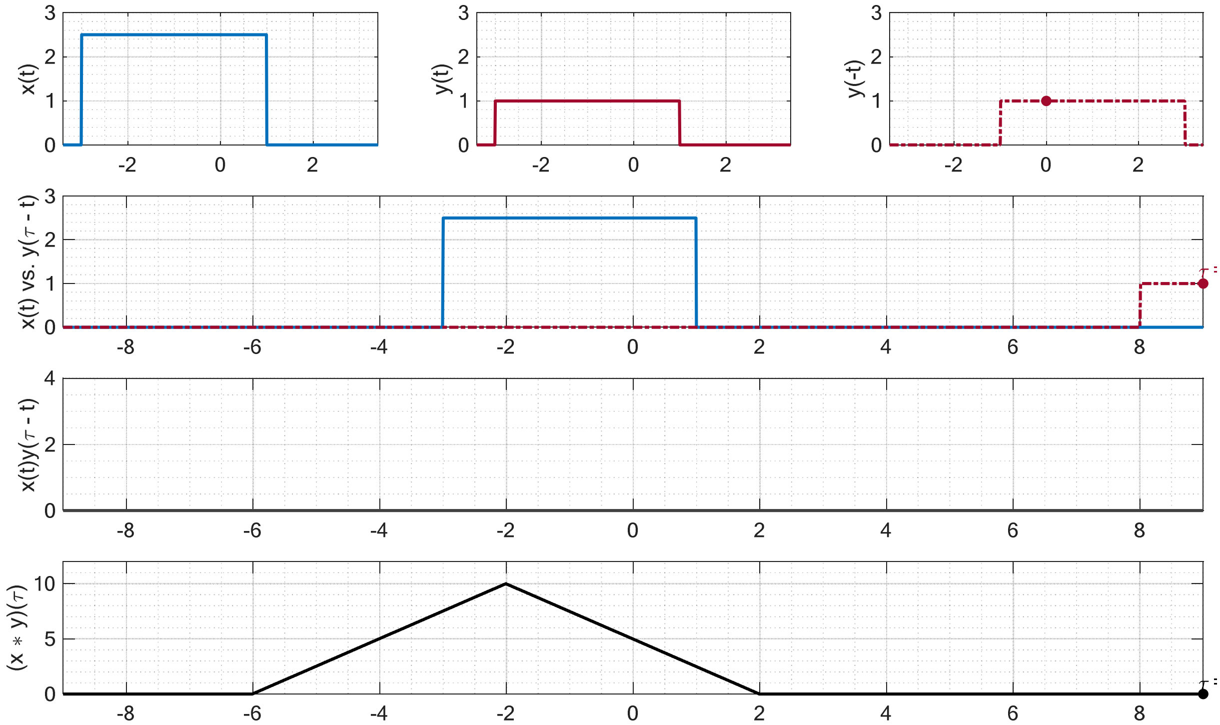
Time convolution property

If we have for two signals $x(t) \Leftrightarrow X(\omega)$ and $y(t) \Leftrightarrow Y(\omega)$, then

$$x(t) * y(t) \Leftrightarrow X(\omega)Y(\omega)$$

Q10. For the signals $x(t) = 2.5 \text{ rect}\left(\frac{t+1}{4}\right)$ and $y(t) = \text{rect}\left(\frac{t+1}{4}\right)$, determine $\mathcal{F}\{x(t) * y(t)\}$.

Q10. Solution. Find $z(\tau) = x(t) * y(t)$ then $\mathcal{F}\{z(\tau)\} = Z(\omega)$, which should be similar to finding $X(\omega)Y(\omega) = Z(\omega)$.



The convolution result is a triangle. Hence, in time-domain:

$$z(t) = x(t) * y(t) = 10\,\Delta\left(\frac{t+2}{4}\right) \equiv z(\tau)$$

$$\mathcal{F}\{\Delta(t)\} = \text{sinc}^2\left(\frac{\omega}{2\pi}\right)$$

$$\mathcal{F}\left\{A\,\Delta\left(\frac{t}{\tau}\right)\right\} = A|\tau|\,\text{sinc}^2\left(\frac{\omega\tau}{2\pi}\right)$$

$$\mathcal{F}\left\{A\,\Delta\left(\frac{t+t_0}{\tau}\right)\right\} = A|\tau|\,\text{sinc}^2\left(\frac{\omega\tau}{2\pi}\right)e^{j\omega t_0}$$

$$Z(\omega) = \mathcal{F}\left\{10\,\Delta\left(\frac{t+2}{4}\right)\right\} = 40\,\text{sinc}^2\left(\frac{4\omega}{2\pi}\right)e^{j2\omega}$$

Alternatively

$$\mathcal{F}\{\text{rect}(t)\} = \text{sinc}\left(\frac{\omega}{2\pi}\right)$$

$$\mathcal{F}\left\{A\,\text{rect}\left(\frac{t}{\tau}\right)\right\} = A|\tau|\,\text{sinc}\left(\frac{\omega\tau}{2\pi}\right)$$

$$\mathcal{F}\left\{A\,\text{rect}\left(\frac{t+t_0}{\tau}\right)\right\} = A|\tau|\,\text{sinc}\left(\frac{\omega\tau}{2\pi}\right)e^{j\omega t_0}$$

$$X(\omega) = \mathcal{F}\left\{2.5\,\text{rect}\left(\frac{t+1}{4}\right)\right\} = 10\,\text{sinc}\left(\frac{4\omega}{2\pi}\right)e^{j\omega}$$

$$Y(\omega) = \mathcal{F}\left\{\text{rect}\left(\frac{t+1}{4}\right)\right\} = 4\,\text{sinc}\left(\frac{4\omega}{2\pi}\right)e^{j\omega}$$

Hence, the other solution technique gives

$$X(\omega)\,Y(\omega) = 10\,\mathrm{sinc}\left(\frac{4\omega}{2\pi}\right)e^{j\omega} \times 4\,\mathrm{sinc}\left(\frac{4\omega}{2\pi}\right)e^{j\omega}$$

$$X(\omega)\,Y(\omega) = 10 \times 4 \times \mathrm{sinc}\left(\frac{4\omega}{2\pi}\right) \times \mathrm{sinc}\left(\frac{4\omega}{2\pi}\right)e^{j(\omega+\omega)}$$

$$X(\omega)\,Y(\omega) = 40\,\mathrm{sinc}^2\left(\frac{4\omega}{2\pi}\right)e^{j2\omega} = Z(\omega)$$

The time convolution property will be very useful when we discuss systems (impulse response).

Frequency convolution property

If we have for two signals $x(t) \Leftrightarrow X(\omega)$ and $y(t) \Leftrightarrow Y(\omega)$, then

$$x(t)\,y(t) \Leftrightarrow \frac{1}{2\pi}X(\omega) * Y(\omega)$$

We consider an example later when discussing sampling where we evaluate $\mathcal{F}\{x(t)\,y(t)\}$, where $y(t) = \mathrm{rep}_{T_s}\{\delta(t)\}$.

Time differentiation property

If we have $x(t) \Leftrightarrow X(\omega)$, then

$$\frac{d}{dt}x(t) \Leftrightarrow j\omega\, X(\omega)$$

$$\frac{d^n}{dt^n}x(t) \Leftrightarrow (j\omega)^n\, X(\omega)$$

Time integration property

If we have $x(t) \Leftrightarrow X(\omega)$, then

$$\int_{-\infty}^{t} x(\tau)d\tau \Leftrightarrow \frac{X(\omega)}{j\omega} + \pi X(0)\,\delta(\omega)$$

Conclusion

Instead of calculating Fourier transform every time using the integral $\int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$, we typically memorize the Fourier transform results for a few basic signals (listed in a table).

Then we find the Fourier transform of any variant of such basic signals by utilizing the appropriate Fourier transform properties (similar to what we do in Laplace transform). The Fourier transform properties are typically also summarized in a table.

Fourier series (complex exponential form) coefficients α_n have similar properties to those of Fourier transform, since you can think of them as the equivalent Fourier transform $2\pi\alpha_n\delta(\omega - n\omega_0)$.

	$\boldsymbol{x(t)}$	$\boldsymbol{X(\omega) = \mathcal{F}\{x(t)\}}$
	$\text{rect}(t)$	$\text{sinc}\left(\frac{\omega}{2\pi}\right)$
	$\Delta(t)$	$\text{sinc}^2\left(\frac{\omega}{2\pi}\right)$
	$\text{sinc}(t)$	$\text{rect}\left(\frac{\omega}{2\pi}\right)$
	$\delta(t)$	1
	1	$2\pi\,\delta(\omega)$
	$u(t)$	$\pi\,\delta(\omega) + 1/(j\omega)$
	$\cos(\omega_0 t)$	$\pi\,\delta(\omega-\omega_0) + \pi\,\delta(\omega+\omega_0)$
	$\sin(\omega_0 t)$	$-j\pi\,\delta(\omega-\omega_0) + j\pi\,\delta(\omega+\omega_0)$
	$\text{sgn}(t)$	$2/(j\omega)$
	$e^{-at}u(t)$	$1/(a+j\omega)$
	$e^{-a\|t\|}$	$2a/(a^2+\omega^2)$
	$\text{rep}_{T_0}\{p(t)\}$, periodic	$\sum_{n=-\infty}^{\infty} 2\pi\alpha_n\delta(\omega-n\omega_0)$
	$\text{rep}_{T_0}\{\delta(t)\}$	$\omega_0\text{rep}_{\omega_0}\{\delta(\omega)\}$

Property	$\boldsymbol{x(t)}$	$\boldsymbol{X(\omega) = \mathcal{F}\{x(t)\}}$
Linearity (superposition)	$a\,x(t) + b\,y(t)$	$a\,X(\omega) + b\,Y(\omega)$
Complex conjugate	$x^*(t)$	$X^*(-\omega)$
Time scaling (reciprocal spreading)	$x\left(\frac{t}{\tau}\right)$	$\lvert\tau\rvert\,X(\tau\omega)$
Time inversion (time reversal)	$x(-t)$	$X(-\omega)$
Time shift (time delay/advance)	$x(t \pm t_0)$	$X(\omega)e^{\pm j\omega t_0}$
Frequency shift	$x(t)e^{\pm j\omega_0 t}$	$X(\omega \mp \omega_0)$
Duality	$X(t)$	$2\pi\,x(-\omega)$
Time convolution	$x(t) * y(t)$	$X(\omega)Y(\omega)$
Frequency convolution	$x(t)\,y(t)$	$\frac{1}{2\pi}X(\omega) * Y(\omega)$
Time differentiation	$\frac{d^n}{dt^n}x(t)$	$(j\omega)^n\,X(\omega)$
Time integration	$\int_{-\infty}^{t} x(\tau)d\tau$	$X(\omega)/(j\omega) + \pi X(0)\,\delta(\omega)$